The ~~Un~~reasonable Effectiveness of Graph Neural Networks for Wireless Communications

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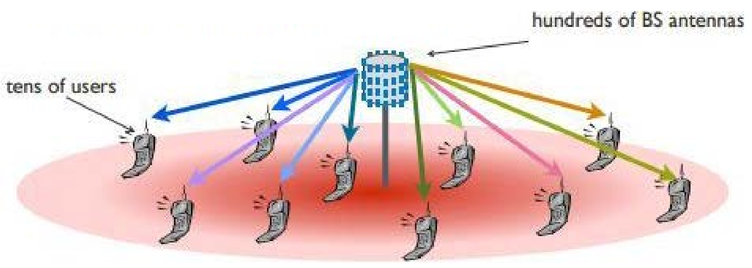
Outline

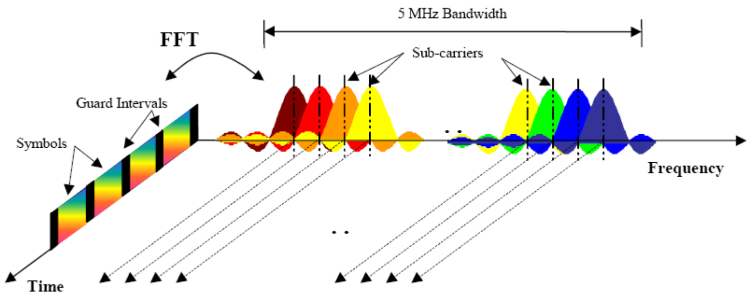
- Computational challenges of wireless networks
- The unreasonable effectiveness of "learn to optimize"
- Graph neural networks (GNNs) for wireless communications
- Theoretical analysis: GNNs vs. MLPs
- Conclusions

Computational challenges of wireless networks

Motivations



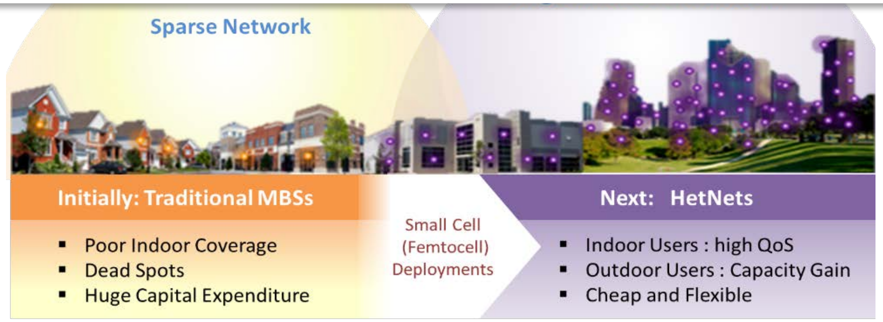
Massive MIMO
(>100 antennas/BS)



OFDM
(>1,000 subcarriers)

Hard large-scale optimization problems are ubiquitous in wireless networks

Massive IoT Access
(millions/km^2)



Ultra-dense Networks
(BS density ~ user density)

Computational challenges in optimizing wireless networks

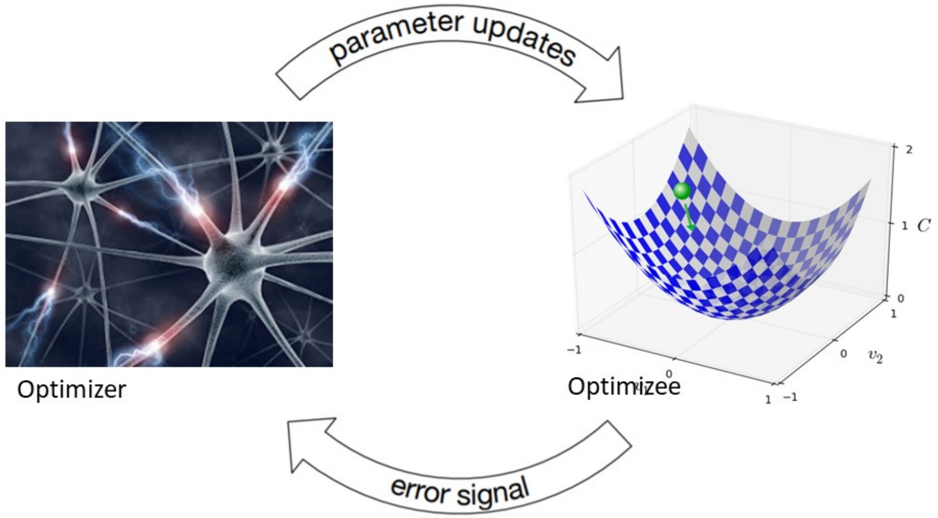
- Enormous problem scales
- Hardness (nonconvexity) of typical problems
- Time-varying channels/topologies (fading/shadowing, mobility)
- Real-time requirement (e.g., AR/VR, UAVs)

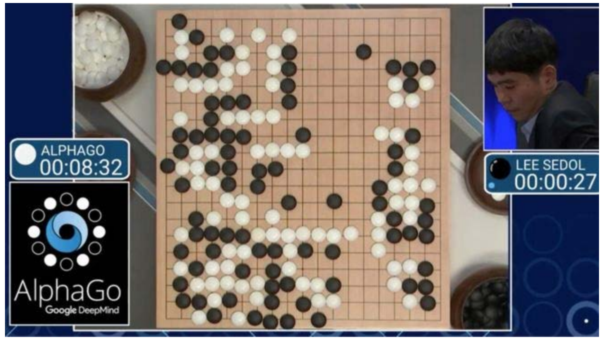
Classic algorithmic approaches

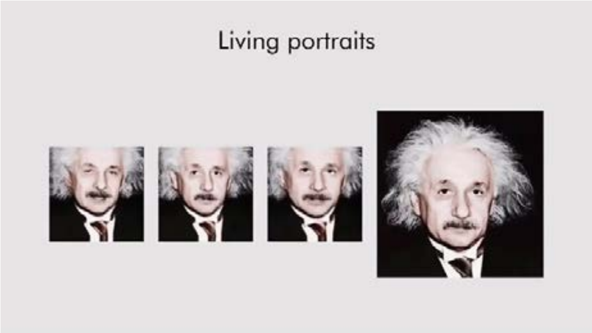
- **Global optimization algorithms**
  - exponential time complexity
  - only work for very small problems

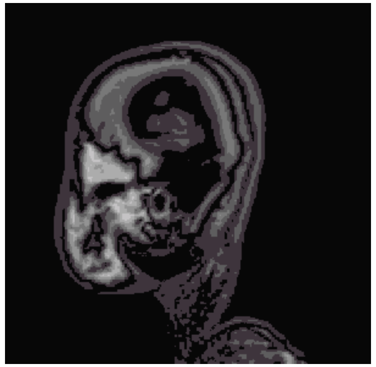
- **Heuristic algorithms**
  - examples: greedy algorithm for user selection
  - hard to design good ones
  - non-negligible gap to the optimal solution
  - difficult to meet real-time requirement

The unreasonable effectiveness of "learn to optimize"





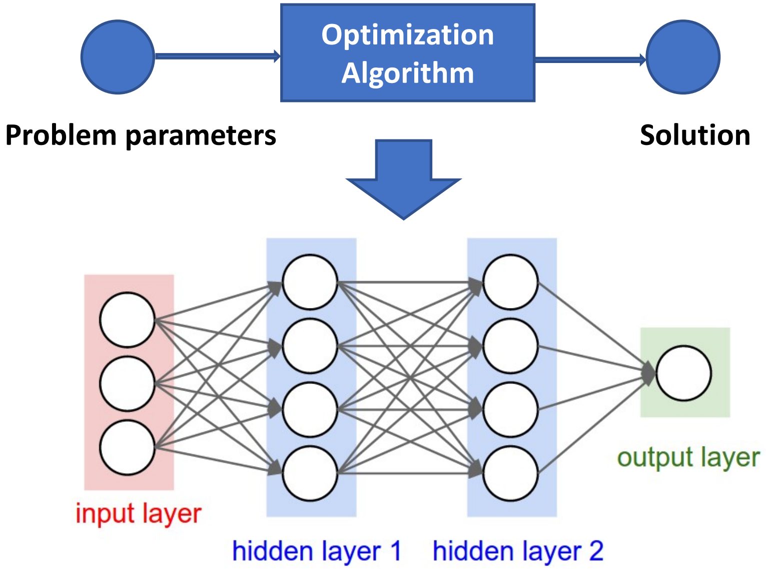




Successes of Deep Learning

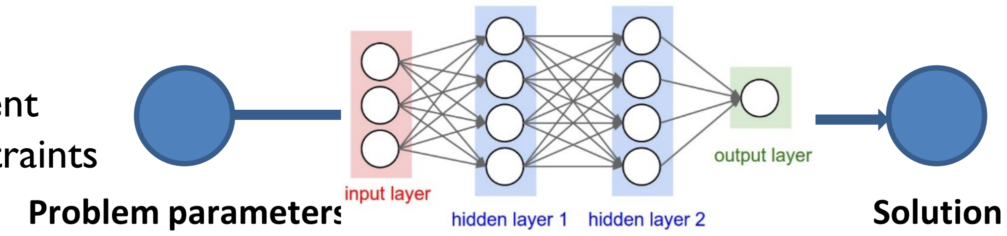
Learning to Optimize

- **Observations**
  - Past data and future problem data have the same distribution
  - Deep learning can learn a good algorithm from data
- **Design Goal**
  - Automatically learn a real-time near-optimal algorithm for difficult optimization problems

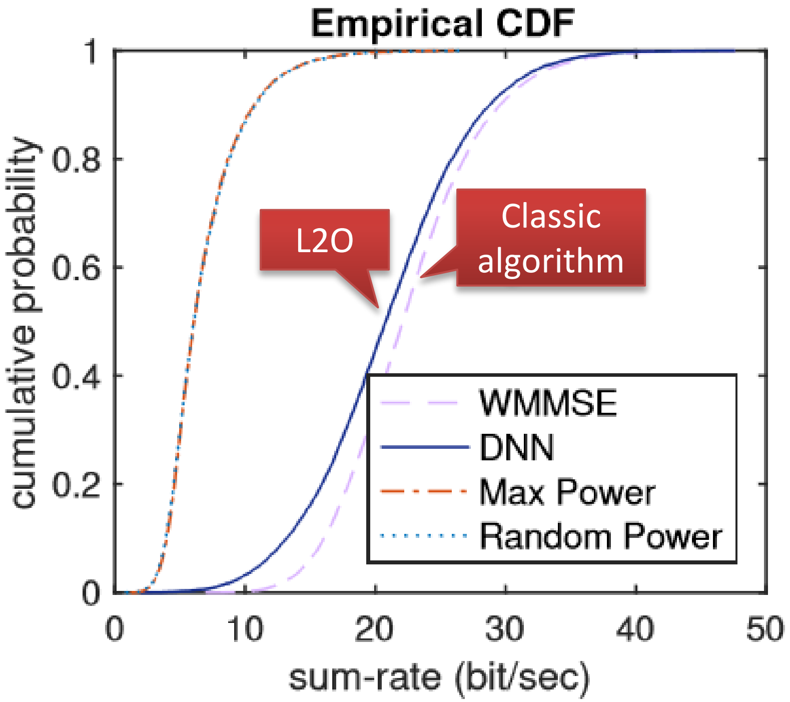


An early attempt

Input
- Channel state
- QoS requirement
- Resource constraints



Output
- Resource allocation
- Detection results



[SCS+18]

- Sum rate maximization of interference channel (NP hard).
- A 3-layer multilayer perceptron (MLP) with [200,80,80] neurons.

[SCS+18] H. Sun, X. Chen, Q. Shi, M. Hong, X. Fu, and N. D. Sidiropoulos, "Learning to optimize: Training deep neural networks for interference management," *IEEE Trans. Signal Process.*, vol. 66, pp. 5438 – 5453, Oct. 2018.

Limitations of L2O via MLPs

- Poor scalability

# of users (K)	average sum-rate (bit/sec.) DNN	WMMSE	DNN/WMMSE
10	2.770	2.817	98.33%
20	3.363	3.654	92.04%
30	3.498	4.150	84.29%

- Huge amounts of samples
  - Millions of samples;
  - Optimal labels are difficult to generate.
- Weak generalization
  - The output dimension of neural networks must be fixed.

Our targets

High sample efficiency

- To be trained with thousands of unlabelled samples

Scalability

- Be able to work for large scale problems

Good generalization

- Be able to generalize to different problem sizes

Theory-guided design principles

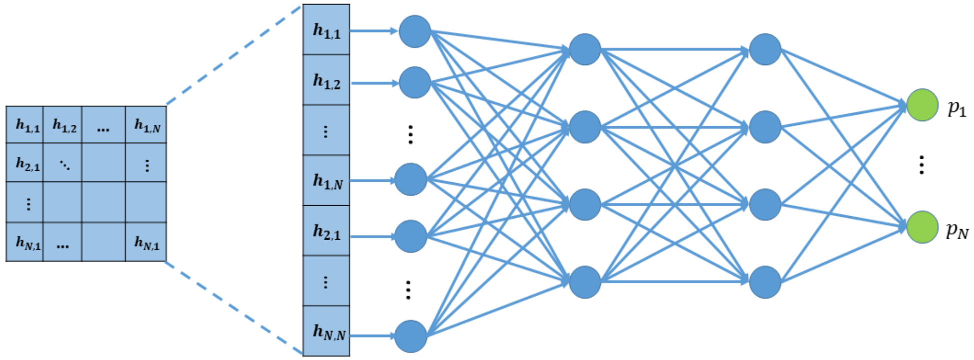
- Effective in designing good neural architecture

Deep Learning: Alchemy or Science?

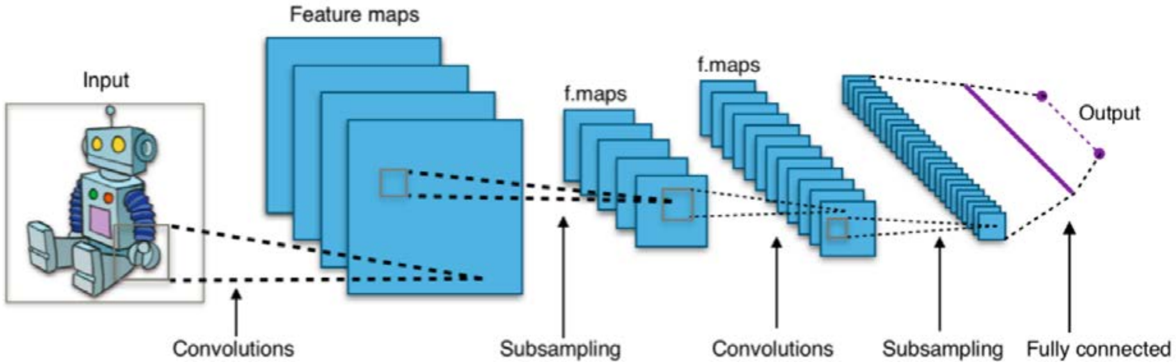
GNNs for wireless communications

Which neural architecture to use?

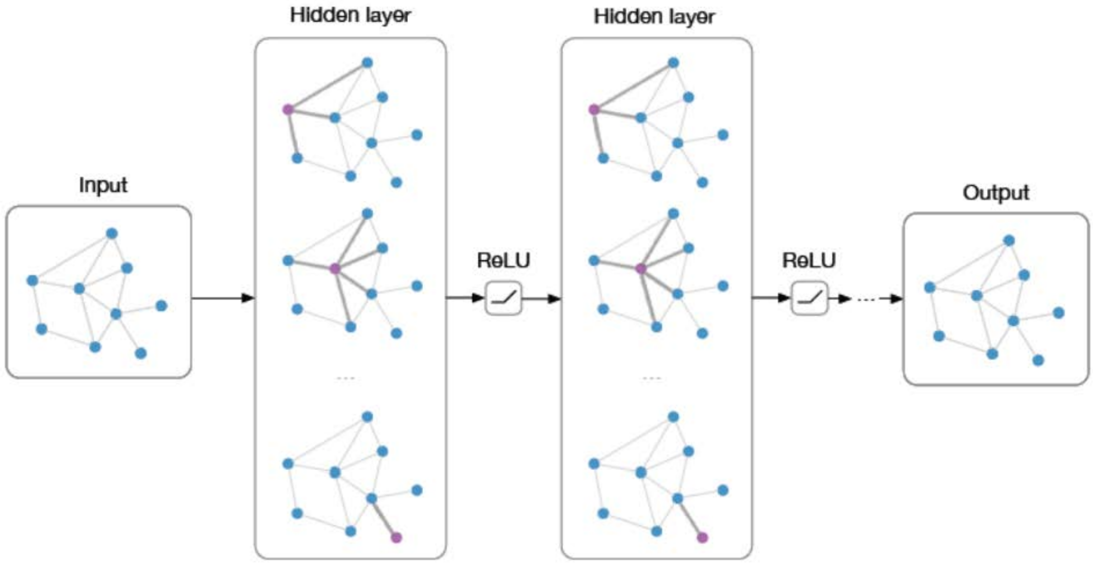
- Why MLP is not effective?
  - It could not exploit structure information in data.



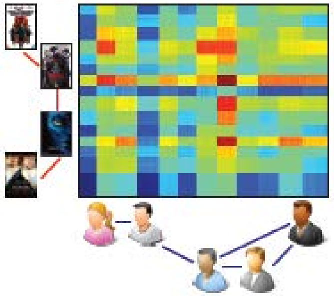
- **A successful story**: Convolutional neural network (CNN) for image processing.
  - It exploits the shift-invariance, local connectivity, compositionality of images.



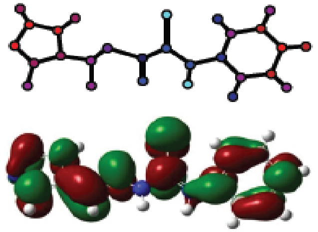
A new architecture: Graph Neural Networks (GNNs)



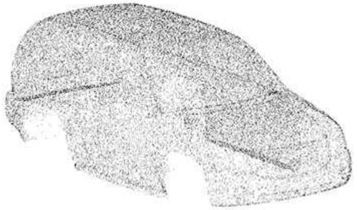
- **GNN Applications**



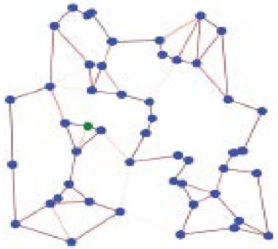
Recommendation



Chemistry



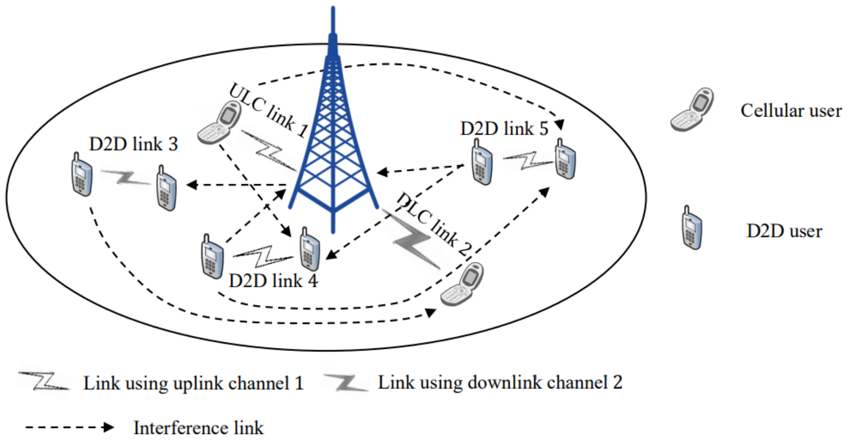
Point clouds

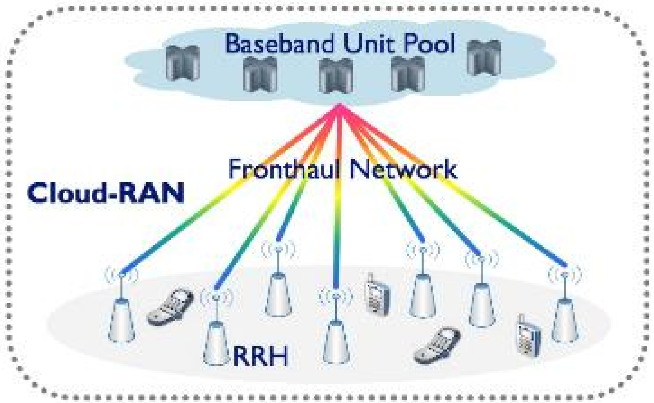


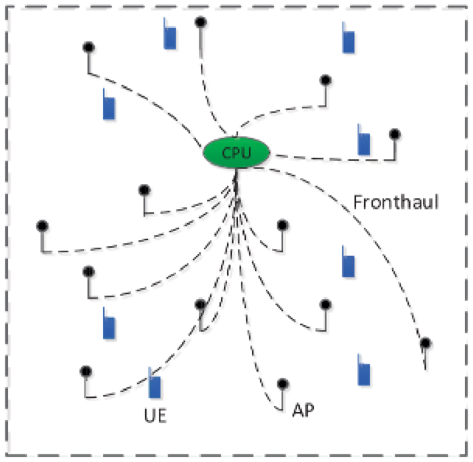
Graph problems

Motivations

- **Observation:** Wireless networks naturally graphs
  - Optimization problems in wireless communication are graph optimization
- **Basic Idea:** Incorporate graph topology into neural networks

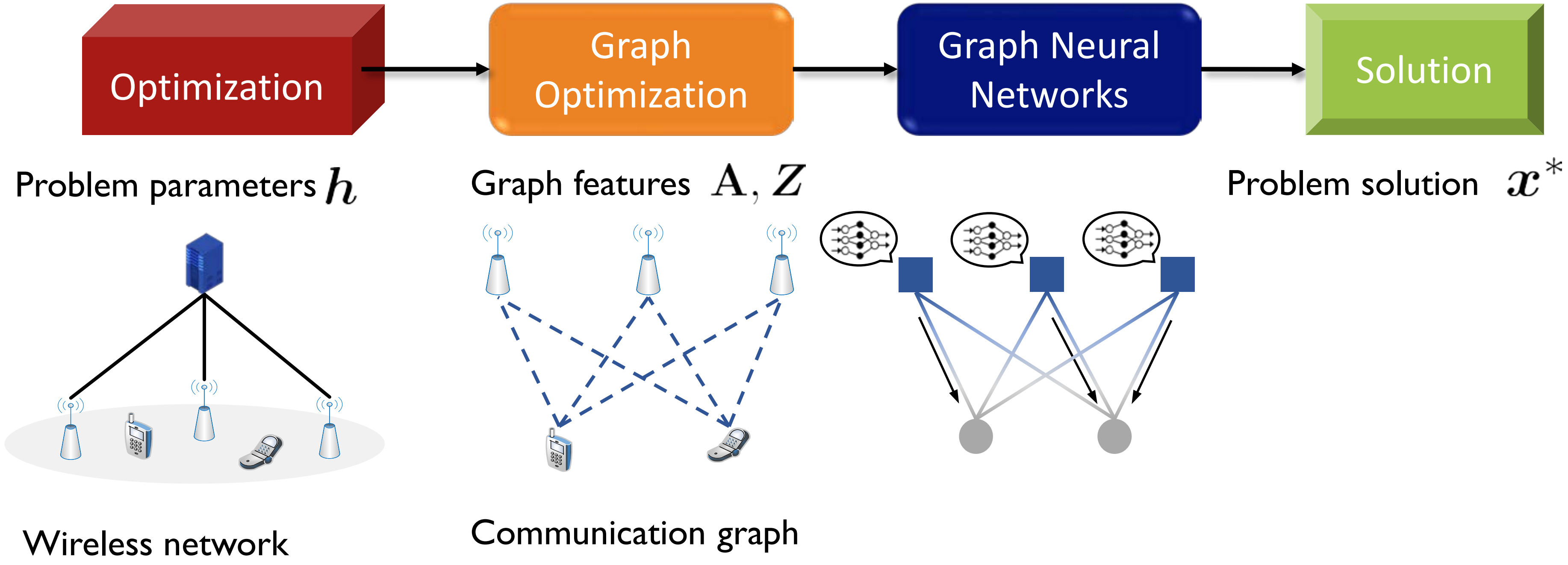






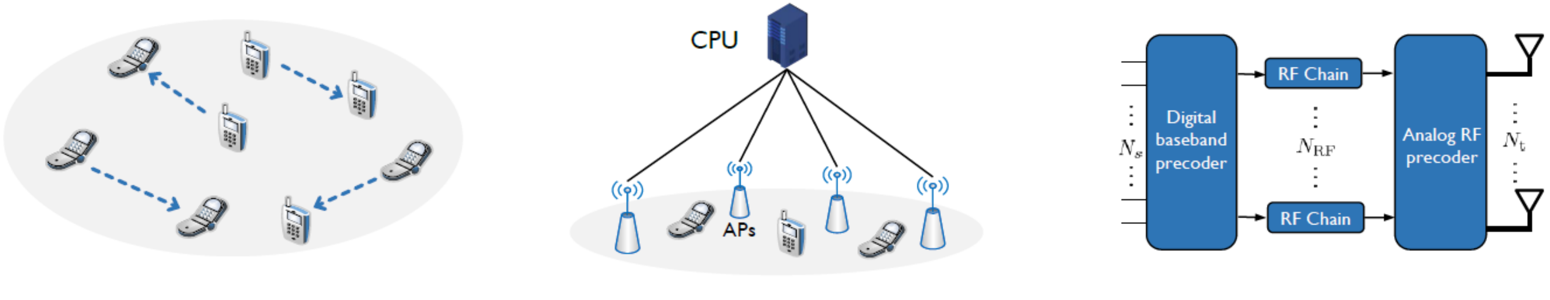
GNN-based framework

- **Proposal:** A two-stage approach for large-scale network optimization

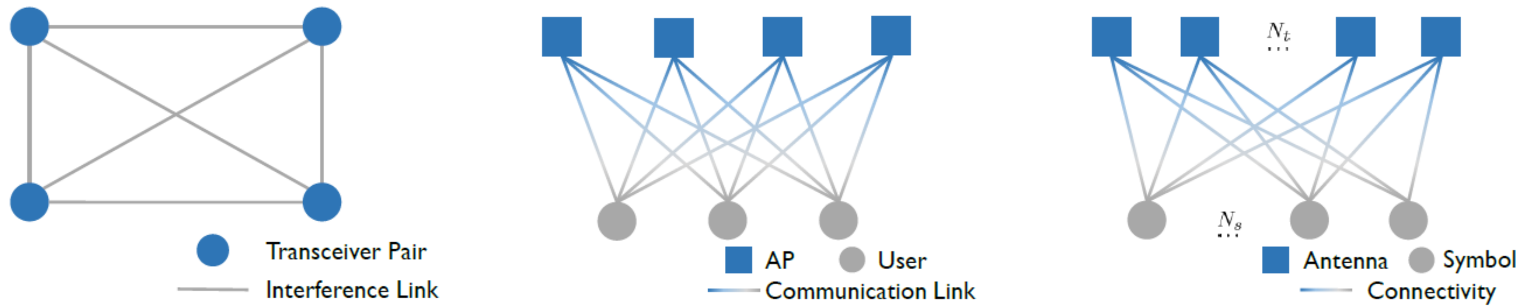


Stage I: Graph modeling

- **Main idea:** network topology as graphs
  - AP/UE as nodes;
  - Communication links as edges
  - Channel information as edge features
- Typical examples: 1) user admission; 1) power control; 2) (hybrid) beamforming

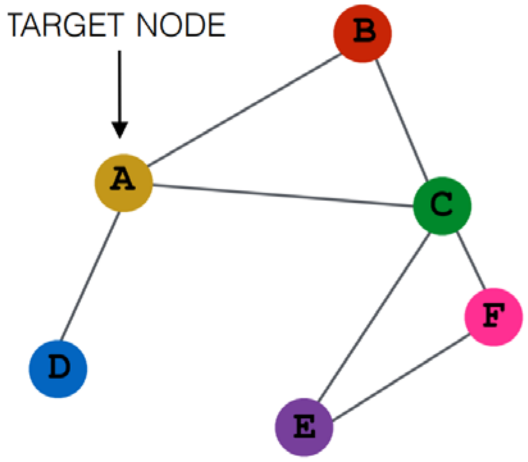


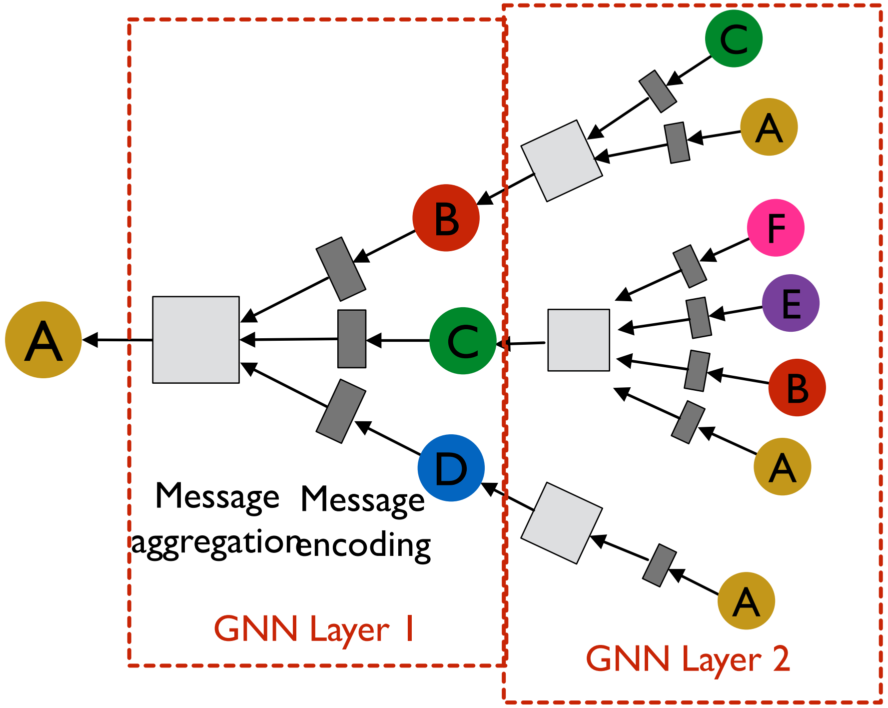
(a) D2D networks. (b) Cell-free networks. (c) Hybrid precoding.



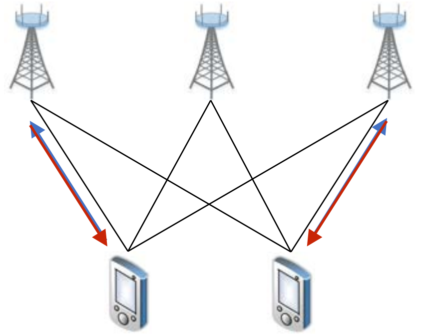
Stage II: Graph neural networks

- **Key idea:** neural message-passing between nodes
- **Two stages:** message encoding & message aggregation





GNNs for wireless networks



($\boldsymbol{V}^{(l)}, \boldsymbol{P}^{(l)}, \boldsymbol{S}^{(l)}, \boldsymbol{U}^{(l)}, \boldsymbol{Q}^{(l)}, \boldsymbol{T}^{(l)}$) are learnable weight matrices

BSs aggregating information from UEs

Learned UE representation

CSI

Message $\mathbf{m}^{(l)}_{\mathrm{UE}\to\mathrm{BS}} = \left(\mathbf{V}^{(l)}\mathbf{u}_j^{(l-1)} + \mathbf{P}^{(l)}\mathbf{h}_{ij}\right)$

Aggregate $\mathbf{b}^{(l)}_{\mathrm{BS}} = \sigma\left(\mathbf{S}^{(l)}\mathbf{b}^{(l-1)}_{\mathrm{BS}} + \sum_{i=1}^{M}\mathbf{m}^{(l)}_{i\to\mathrm{BS}}\right)$

Learned BS representation vector

UEs aggregating information from BSs

Learned BS representation

CSI

Message $\mathbf{m}^{(l)}_{\mathrm{BS}\to\mathrm{UE}} = \left(\mathbf{U}^{(l)}\mathbf{b}_j^{(l-1)} + \mathbf{Q}^{(l)}\mathbf{h}_{ji}\right)$

Aggregate $\mathbf{u}^{(l)}_{\mathrm{UE}} = \sigma\left(\mathbf{T}^{(l)}\mathbf{u}^{(l-1)}_{\mathrm{UE}} + \sum_{i=1}^{K}\mathbf{m}^{(l)}_{i\to\mathrm{UE}}\right)$

Learned UE representation vector

Why GNN? – Theoretical support

- A class of distributed algorithms called *distributed message passing (DMP)*, including many classic algorithms
  - Fractional programming for power control [Shen18TSP]
  - WMMSE for beamforming [Shi11TSP]
  - Riemannian gradient for hybrid precoding [Yu16JSTSP]

- **Theorem:** equivalence between GNNs and DMP algorithms
  1. GNNs are special cases of DMP algorithms
  2. For any DMP algorithm, there exists a GNN that approximates it well

Why GNN? – Theoretical support

- **Theorem:** equivalence between GNNs and DMPs
  1. GNNs are special cases of DMPs
  2. For any DMP algorithm, there exists an GNN that approximate it well

- **Interpretations:**
  1. The distributed property allows GNNs to achieve:
     - **Good generalization**: To generalize to any number of UEs/APs during the test
     - **High computation efficiency**: To have constant running time independent of number of UEs/APs
  2. If a GNN is trained well, its performance is at least as good as DMPs

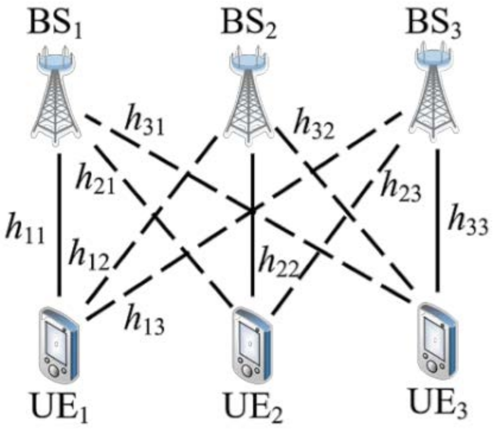
Case study: Power control

- **Sum rate maximization in K-user interference channel**
  - h_{kk} is the direct channel of k-th user
  - h_{kj} is the interference channel between j-th and k-th user
  - p_k is the power for k-th transmitter

- **NP-hard**

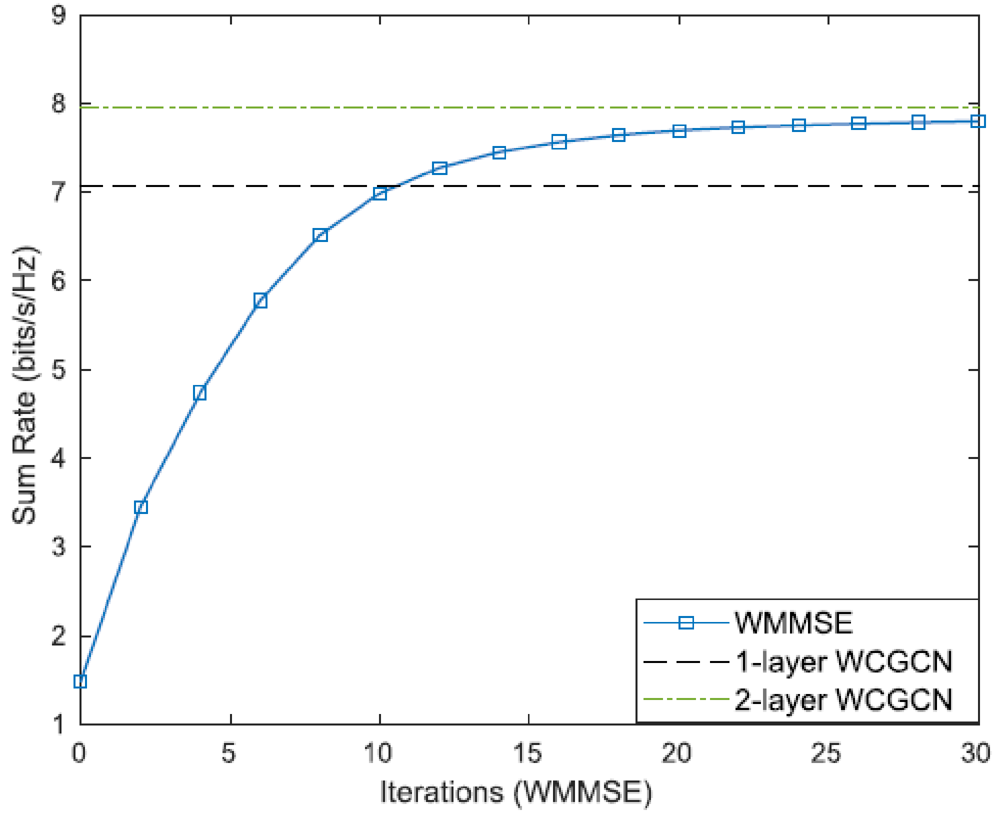
$$\begin{aligned} \underset{p_1,\cdots,p_K}{\text{maximize}} \quad & \sum_{k=1}^{K} \log_2\left(1+\frac{|h_{kk}|^2 p_k}{\sum_{j\neq k}|h_{kj}|^2 p_j+\sigma_k^2}\right) \\ \text{subject to} \quad & 0 \le p_k \le 1. \end{aligned}$$

We adopt unsupervised training, i.e., no label is needed!



Simulations: GNN vs. Distributed Algorithm

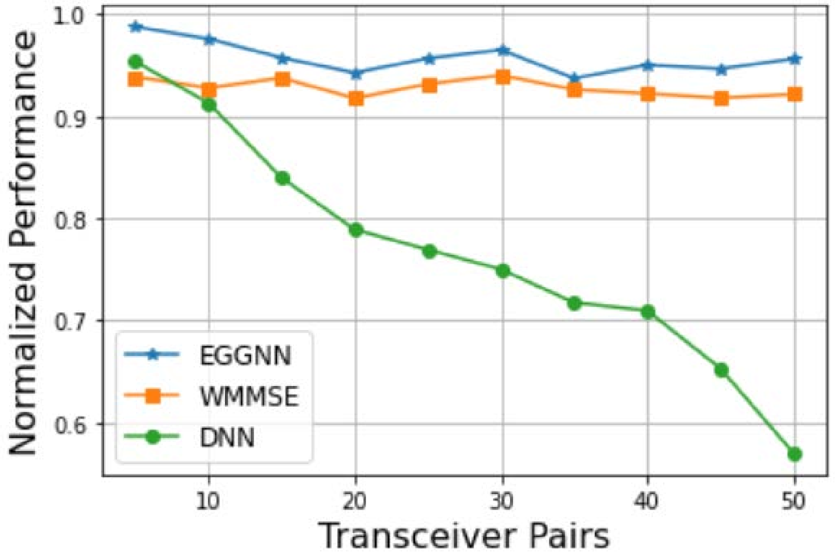
- A 1-layer GNN outperforms WMMSE (it belongs to DMP) with 10 iterations.
- A 2-layer GNN outperforms WMMSE with 30 iterations.



Simulations: Improved performance

- **Legends**
  - EGGNN: Proposed GNN method
  - WMMSE: a widely adopted optimization-based method [Shi11TSP]
  - DNN: DNN-based method [Sun18TSP]
- **Advantages**
  - Near-optimal performance, better than WMMSE
  - Much better scalability than DNN

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \sum_{k=1}^{K} \log_2 \left(1 + \frac{|h_{k,k}|^2 p_k}{\sum_{j \neq k}^{K} |h_{j,k}|^2 p_j + \sigma_k^2} \right), \\ \text{subject to} \quad & 0 \leq p_k \leq P \end{aligned}$$

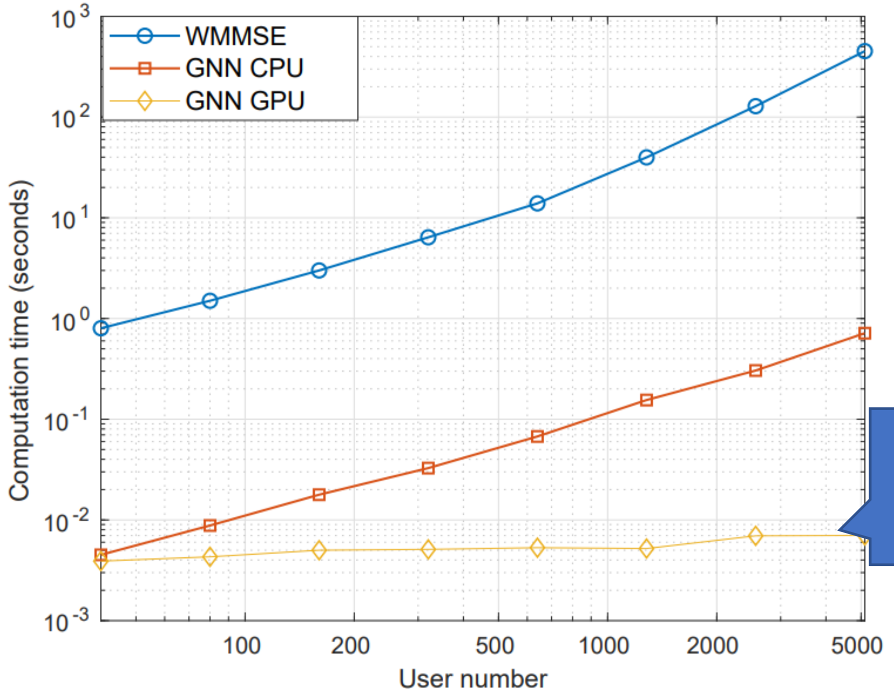


Performance is normalized by the best of FPlinQ with 100 different initialization points

Simulations: Scalability

- **Setting:** GNN trained on 50 users and test on different numbers of users

- **Advantages**
  - Orders of magnitude of speedups
  - On CPU, computation time is reduced by 100~1000 times compared with WMMSE
  - Nearly constant time scale up on GPU



6 ms for thousands of users;
100,000x reduction!

Simulations: Generalization

- **Generalization to larger scales**
  - Trained with 50 pairs in a 1000m×1000m region.
  - Test with different numbers of pairs while the density of users is fixed.

GENERALIZATION TO LARGER PROBLEM SCALES BUT SAME DENSITY

Links	Size (m^2)	($d_{\min}, d_{\max}$)	
		(10m,50m)	(30m,30m)
200	2000×2000	98.3%	98.1%
400	2828×2828	98.9%	98.2%
600	3464×3464	98.8%	98.7%
800	4000×4000	98.9%	98.6%
1000	4772×4772	98.9%	98.7%

- **Generalization to higher densities**
  - Trained with 50 pairs in a 1000m×1000m region.
  - Change the number of pairs in the test set while fixing the area size.

GENERALIZATION OVER DIFFERENT LINK DENSITIES. THE PERFORMANCE LOSS COMPARED TO $K = 50$ IS SHOWN IN THE BRACKET

Links	Size (m^2)	($d_{\min}, d_{\max}$)	
		(10m,50m)	(30m,30m)
100	1000×1000	97.6% (+0.1%)	96.4% (−0.1%)
200		97.0% (−0.5%)	96.0% (−0.5%)
300		95.9% (−1.6%)	94.9% (−1.6%)
400		95.6% (−1.9%)	94.5% (−2.0%)
500		95.3% (−2.2%)	94.5% (−2.0%)

Theoretical analysis: GNN vs. MLP

The reasonable effectiveness of GNNs

Theoretical analysis

- So far, we have shown
  - Incorporating prior knowledge of wireless communications into neural architectures improves performance

- **Goal of this part:** to theoretically characterize
  - *How many training samples* are needed to train the neural network well?
  - *How much performance gains* can "structure" bring?

Generalization analysis of deep learning

- **Generalization Analysis via PAC-learning**
  - Test data are drawn i.i.d. from unknown distribution *D;*
  - $f^*(\cdot)$ is an oracle algorithm generating an optimal solution;
  - $f(x_i, W^*)$ is neural network where *W* is weight obtained by training;
  - Given *N* training samples, if with probability at least δ, we have

$$\mathbb{E}_{x\sim\mathcal{D}}\|f(x, W^*) - f^*(x)\| \leq \epsilon$$

  - then we call $f^*(\cdot)$ is (ε, δ, N) learnable by this neural network.
  - The minimum number of training samples *N* is called the sample complexity

- **Key idea (algorithmic alignment)**
  - If the neural network shares a common structure with the oracle function $f^*(\cdot)$, then the sample complexity is low.

Keyulu Xu, Jingling Li, et. al, "What can neural networks reason about?" ICLR 2020.

Main result

- Target Example: GNNs versus unstructured deep neural networks (DNNs), i.e., MLP, for "learning to optimize" to solve large-scale graph optimization problems.

- **Theorem:** Consider an optimization problem on a |V|-node graph.
  - For GNNs, it is $(\epsilon, \delta, \mathcal{O}(C_{\mathcal{A}}(f_0, \cdots, f_m)/|V|))$-learnable.
  - For MLPs, it is $(\epsilon, \delta, \mathcal{O}(C_{\mathcal{A}}(f_0, \cdots, f_m)|V|))$-learnable.
    - $C_{\mathcal{A}}(f_0, \cdots, f_m)$ is a constant related to
      1. maximum degree of the graph;
      2. optimization objectives;
      3. NN substructure, e.g., activation.

Proof sketch

- **Lemma 1:** For any graph optimization problem, there exists a distributed message passing algorithm that can solve it.
- **Lemma 2:** If the neural network *A* can simulate the task algorithm with *n* modules, and each module is $(\varepsilon, \delta, M/n)$ learnable, then the task is (ε, δ, M) learnable by *A*.
- **Lemma 3:** To simulate a DMP algorithm,
  - GNNs require $\mathcal{O}(1)$ modules that are $(\epsilon, \delta, O(C/|V|))$ learnable;
  - MLPs require $\mathcal{O}(|V|)$ modules that are $(\epsilon, \delta, O(C))$ learnable.

GNNs aligns with DMP algorithms better than MLPs

Corollary: GNNs vs. MLPs

- To learn DMP algorithms to solve a graph optimization problem.
  - For GNNs, it is $(\epsilon, \delta, \mathcal{O}(C_{\mathcal{A}}(\psi, f_0, \cdots, f_m)/|V|))$-learnable
  - For MLPs, it is $(\epsilon, \delta, \mathcal{O}(C_{\mathcal{A}}(\psi, f_0, \cdots, f_m)|V|))$-learnable

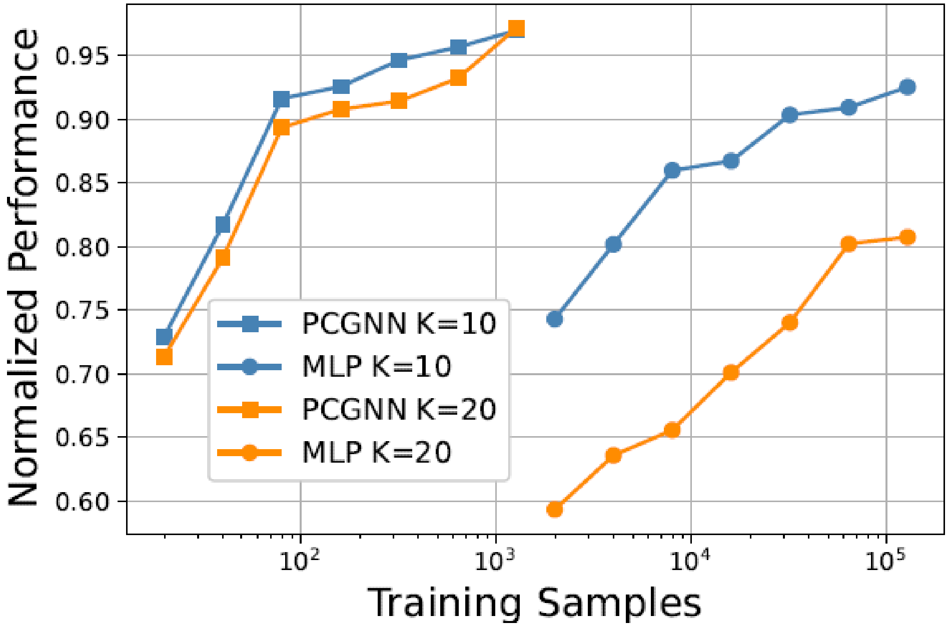
→ GNNs vs. MLPs (sample complexity)

- GNNs require $\mathcal{O}(|V|^2)$ times fewer training samples than MLPs.
- GNNs' performance gap to the optimal solution is $\mathcal{O}(|V|)$ times lower than MLPs (see our paper for more details).

Simulations: Sample complexity

- We consider K-user interference channel power control.
- **Theory:** MLPs require $\mathcal{O}(|V|^2)$ times more training samples than GNNs.
- **Simulations**
  - For 10 users, MLPs require 100 times more training samples;
  - For 20 users, MLPs require 400~600 times more training samples.

$$\begin{aligned} \underset{\boldsymbol{p}}{\text{maximize}} \quad & \sum_{k=1}^{K} \log_2 \left(1 + \frac{|h_{k,k}|^2 p_k}{\sum_{j \neq k}^{K} |h_{j,k}|^2 p_j + \sigma_k^2} \right), \\ \text{subject to} \quad & 0 \leq p_k \leq P \end{aligned}$$



Performance is normalized by the best of FPlinQ with 100 different initialization points

Corollary: Stable performance of GNNs

- Consider a graph optimization problem on graph $G = (V, E)$
  - For GNNs, it is $(\epsilon, \delta, \mathcal{O}(C_{\mathcal{A}}(\psi, f_0, \cdots, f_m)/|V|))$-learnable
    - $C_{\mathcal{A}}(\psi, f_0, \cdots, f_m)$ is a constant related to 1) maximum degree of the graph; 2) optimization objectives; 3) NN substructure, e.g., activation.

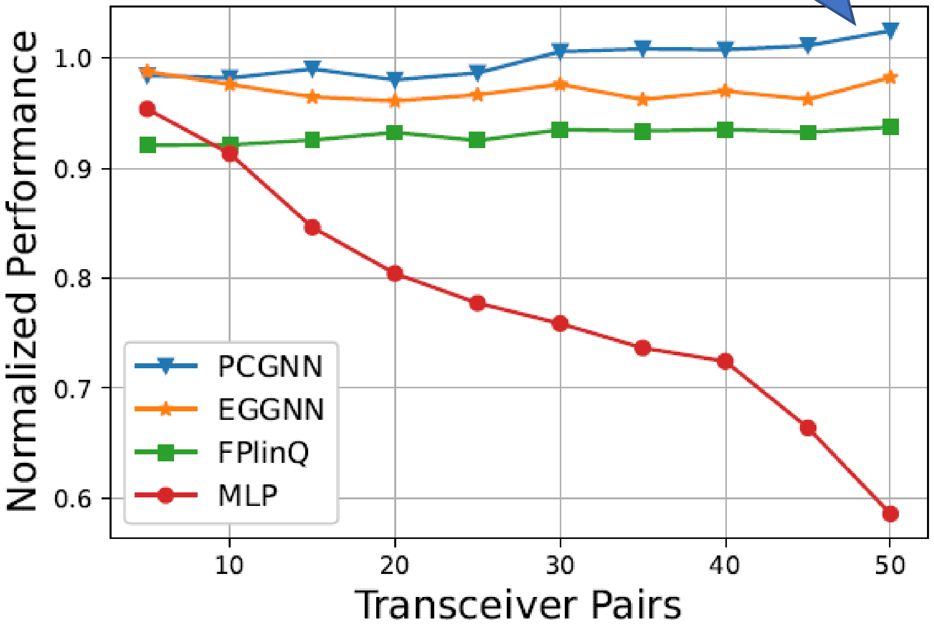
Stable performance of GNNs

- If the maximum degree does not change, the performance of GNNs is stable and independent of the node (user) number;
- Neural architectures can be specially designed to improve the bound.

Simulations: Neural architecture design

- **Legends**
  - **PCGNN**: GNN designed to optimize the bound
  - EGGNN: [Shen20JSAC]
  - FPlinQ: optimization-based method (fractional programming)
  - MLP: MLP-based method [Sun18TSP]
- **Simulations**
  - PCGNN shows consistently better performance than ECGNN.

PCGNN outperforms Best FPlinQ (among 100 trials)



Performance is normalized by the best of FPlinQ with 100 different initialization points

Conclusions

Conclusions

- GNNs for wireless communications
  - Good performance (beat SOTA algorithms)
  - Good generalization (to different problem sizes)
  - High computational efficiency (orders of magnitude speedup)
  - Wide applications
- The **reasonable** effectiveness of GNNs
  - The common structure in the target task and neural network improves sample efficiency and performance
  - Supported quantitively via PAC-learning theory + algorithmics alignment
- For reproducibility
  - https://github.com/yshenaw/GNN4Com

Future directions

- **The picture is far from complete!**
- Extend to other applications
  - GNNs for channel estimation
  - GNNs for MIMO detection
  - …
- Extend to other learning approaches
  - GNNs with model-driven deep learning
- Further improve robustness to distribution shift
  - To improve out of distribution (OOD) generalization

ROBUSTNESS TO THE CHANGE OF CHANNELS. $K = 10$.

Setting	User distribution shift	Antenna height distribution shift	LoS $\rightarrow$ NLoS	ITU $\rightarrow$ LTE
PCGNN	97.62%	96.90%	94.43%	88.71%
PCGNN (Full Training)	97.78%	97.63%	97.53%	96.57%
FPlinQ	93.51%	93.66%	93.51%	94.32%

References

- H. Sun, X. Chen, Q. Shi, M. Hong, X. Fu, and N. D. Sidiropoulos, "Learning to optimize: Training deep neural networks for interference management," *IEEE Trans. Signal Process.*, vol. 66, pp. 5438 – 5453, Oct. 2018.
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Thank you!

- For more details

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